

STAT

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MEMORANDUM

Declass Review by NGA.

To:
From:
Subject: On the Relationship Between Sine Wave Modulation and
Some Coherent Optical Filtering Measurements
CC:

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INTRODUCTION

In order to evaluate microdensitometers, it is desirable to have an alternate method for determining the modulation, M , of intensity transmittance recorded on film as

$$T = A (1 + M \sin \omega_0 x) \quad (1)$$

It has been proposed elsewhere¹ that the modulation can be determined by measurements on the spectrum of the corresponding amplitude transmittance, t , where

$$t = \sqrt{T} = \sqrt{A} (1 + M \sin \omega_0 x)^{1/2} \quad (2)$$

as obtained using coherent light. For small M , we have

$$t \approx \sqrt{A} \left(1 + \frac{M}{2} \sin \omega_0 x \right) \quad (3)$$

and it would appear that M can be determined from the ratio*

$$\lambda = \frac{\text{Intensity of Fundamental}}{\text{Intensity of DC}} = \frac{A \frac{M^2}{4}}{A} = \frac{M^2}{4} \quad (4)$$

* It should be noted that whereas the analysis herein uses single sided spectra, coherent optics displays the two sided spectrum. This means that only the intensity of one term of $\sin \omega_0 x = \frac{1}{2} e^{i\omega_0 x} + \frac{1}{2} e^{-i\omega_0 x}$ can be measured so that $M/4$ becomes $M/16$ as far as practical measurements are concerned.

It is of interest to determine the range of M for which the above approximate relationship is valid by considering a more exact expression such as might be obtained from the expansion of (2) in a Fourier series.

FOURIER SERIES

Without loss of generality, we can take $A=1$ and write

$$t = (1 + M \sin \omega_0 x)^{1/2} = \frac{a_0}{2} + \sum_{n=1}^{\infty} (a_n \cos n \omega_0 x + b_n \sin n \omega_0 x) \quad (5)$$

Instead of using the standard integral formulas for the coefficients, a_n and b_n , it is easier to proceed using certain known expansions. From the binomial formula²

$$(1+z)^p = \sum_{n=0}^{\infty} \binom{p}{n} z^n \quad (6)$$

valid for $|z| < 1$ and p any real number,* we obtain

$$t = \sum_{n=0}^{\infty} \binom{1/2}{n} M^n \sin^n \omega_0 x \quad (7)$$

There are two expansions³ for $\sin^n \omega_0 x$, one for n even, the other for n odd. These are

$$\sin^{2k} \theta = \frac{(-1)^k}{2^{2k-1}} \left\{ \sum_{j=0}^{k-1} (-1)^j \binom{2k}{j} \cos 2(k-j) \theta + (-1)^k \frac{1}{2} \binom{2k}{k} \right\} \quad (8)$$

and

$$\sin^{2k+1} \theta = \frac{(-1)^k}{2^{2k}} \sum_{j=0}^k (-1)^j \binom{2k+1}{j} \sin (2[k-j]+1) \theta \quad (9)$$

Substituting (8) and (9) in (7) and comparing with (5), we find that $a_1 = 0$ and

$$\begin{aligned} a_0 &= \sum_{k=0}^{\infty} \alpha_{2k} M^{2k} ; & \alpha_{2k} &= \frac{1}{2^{2k}} \binom{1/2}{2k} \binom{2k}{k} \\ b_1 &= M \sum_{k=0}^{\infty} \beta_{2k} M^{2k} ; & \beta_{2k} &= \frac{1}{2^{2k}} \binom{1/2}{2k+1} \binom{2k+1}{k} \end{aligned} \quad (10)$$

*

$\binom{p}{n} = \frac{p(p-1)(p-2)\dots(p-n+1)}{n!}$
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Consequently, we can write for λ (as defined in 4) an exact expression, viz.

$$\lambda = \frac{b_1^2}{a_0^2} = M^2 R^2 \quad (11)$$

In which

$$R = \frac{\sum_{k=0}^{\infty} \beta_{2k} M^{2k}}{\sum_{k=0}^{\infty} \alpha_{2k} M^{2k}} = \sum_{k=0}^{\infty} \gamma_{2k} M^{2k} \quad (12)$$

The coefficients γ_{2k} are determined from the recurrence relationships

$$\beta_{2k} = \sum_{n=0}^k \gamma_{2n} \alpha_{2k-2n} ; \quad k = 0, 1, 2, \dots \quad (13)$$

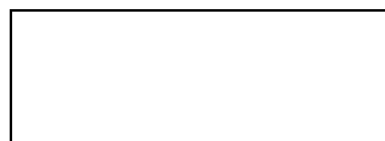
Solving (13) for γ_0 , γ_2 , γ_4 , and γ_6 , we find that (11) becomes

$$\lambda = \frac{M^2}{4} \left\{ 1 + \frac{5}{32} M^2 + \frac{15}{256} M^4 + \frac{1,965}{65,536} M^6 + \dots \right\}^2 \quad (14)$$

Comparing (14) with (4), we see that the first correction term $\frac{5}{32} M^2$ is about 3% of 1 for 50% modulation and would change λ by 6%.

REMARKS

Additional studies of the harmonic distortion introduced by non-linear operations could be based on the techniques referred to in the bibliography of Reference 4.



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